

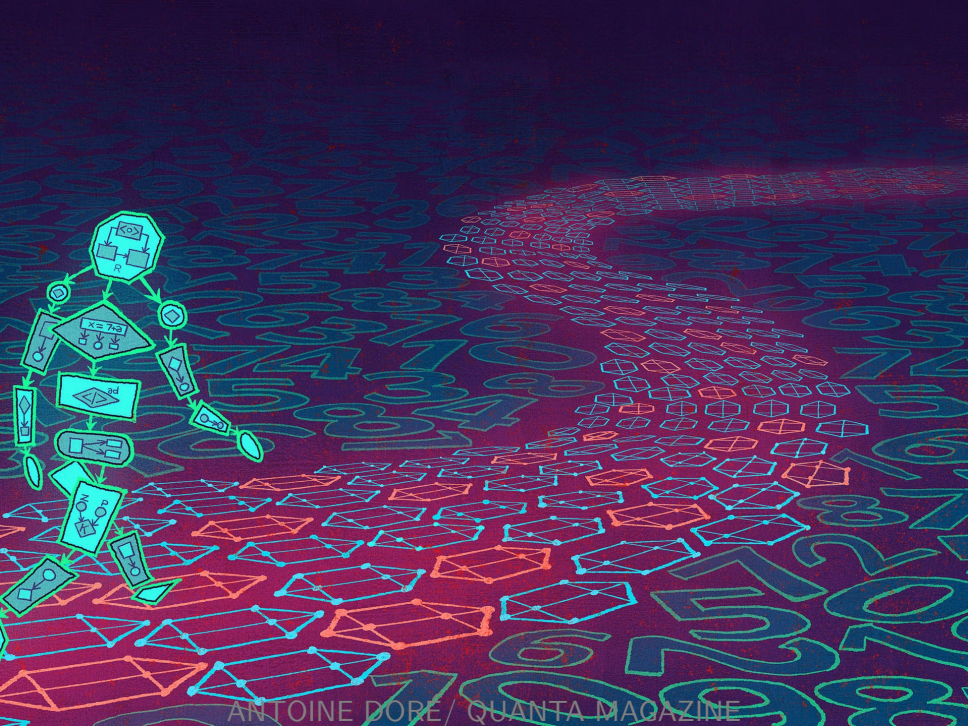
# Topics in TCS

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**An introduction to data streaming**

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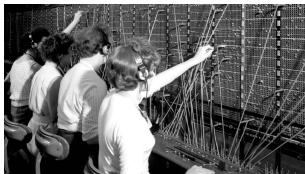
**Raphaël Clifford**



ANTOINE DORÉ / QUANTA MAGAZINE

# Data streaming

This unit is about algorithms for processing data streams. We will develop fast, small space data, typically but not always randomised data structures and algorithms.



For a small subset of the many applications, see e.g. Google's page on the Count-Min sketch<sup>1</sup>.

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<sup>1</sup>Some of the links are broken unfortunately but the application links work

## What is in this half of the unit?

Subject:	Topics	Reference
What is streaming?	Introduction	
Probability overview	Markov, Chebyshev, Chernoff	MIT notes
Finding frequent elements	The Misra-Gries algorithm	Ch. 1
Counting distinct elements	The Tidemark algorithm	Ch. 2
Counting part II	The BJKST algorithm	Ch. 3
Approximate counting	The Morris counter	Ch. 4
Finding frequent items	CountSketch/Min Sketch	Ch. 5
Sparse recovery	Fingerprinting and hashing	Ch. 9
$\ell_0$ -sampling	Sample by frequency	Section 10.2

The set text is the Data Stream Algorithms by Chakrabati. A version without the word DRAFT is linked from the unit web page.

## What is in the second half of the unit?

Subject:	Topics	Reference
Graph streams?	Connectivity, Bipartiteness	Ch. 14.2, 14.3
Shortest distances	Computing spanners	Ch. 14.4
Matchings I	Unweighted and weighted	Ch. 15
Matchings II	Multiple passes	Sec. 3 in [1]
Matchings III	Insertion-deletion streams	Sec. 5 in [2]
The AGM sketch	Connectivity with deletions	Ch. 16
Lower Bounds	Communication complexity	Ch. 18
Lower Bounds II	Yao's Lemma, INDEX problem	Ch. 18
Lower Bounds III	Further reductions	Ch. 18

[1] S. Kale, S. Tirodkar: Maximum Matching in Two, Three, and a Few More Passes Over Graph Streams. APPROX-RANDOM 2017.

<https://arxiv.org/pdf/1702.02559.pdf>

[2] C. Konrad: Maximum Matching in Turnstile Streams. ESA 2015.

<https://arxiv.org/pdf/1505.01460.pdf>

# What is data streaming?



IP	Frequency
37.56.181.226	5
241.79.159.27	1
163.0.199.170	13
62.26.98.238	0
47.127.134.141	4
4.232.47.134	3
16.13.141.93	7

# What is data streaming?

How many distinct IPs? What is the most frequent IP?  
Estimate frequency of an IP? Randomly sample an IP. Etc.



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small space

one-pass

## Streaming graphs

*s*

*b*

*c*

*d*

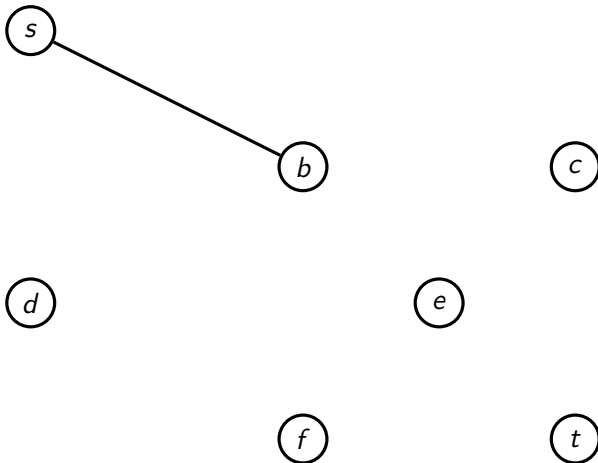
*e*

*f*

*t*

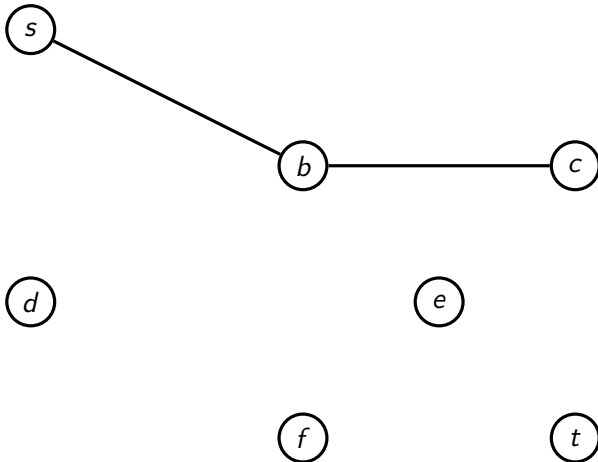
# Streaming graphs

Add edge



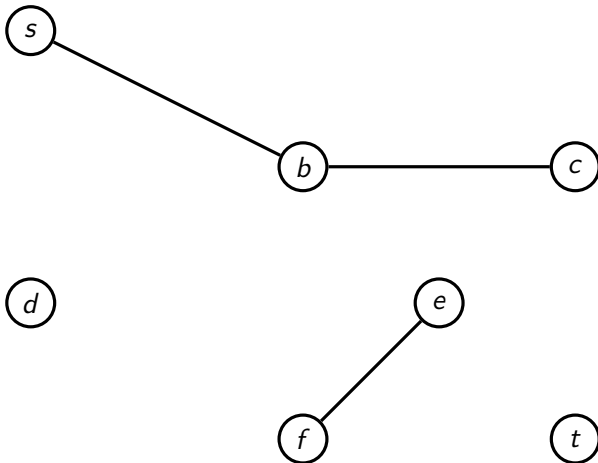
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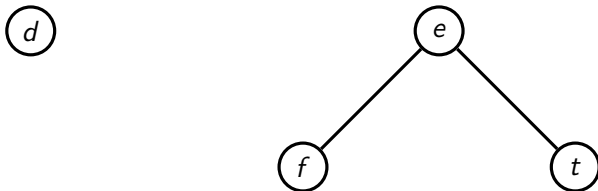
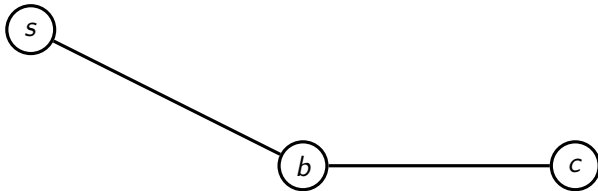
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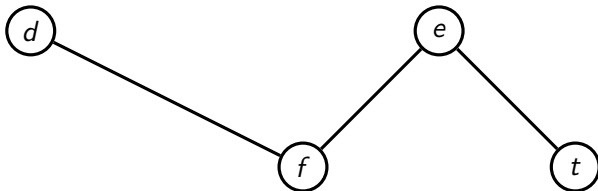
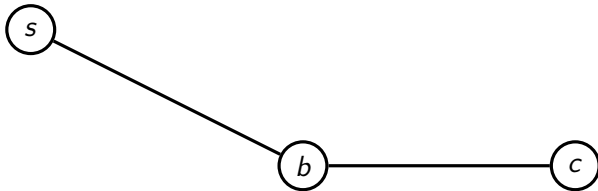
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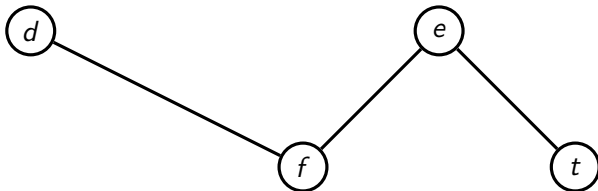
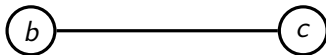
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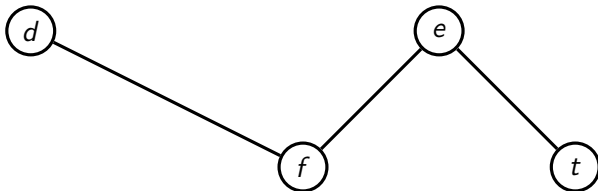
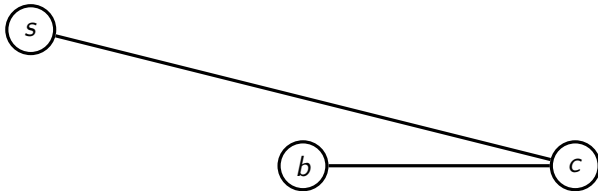
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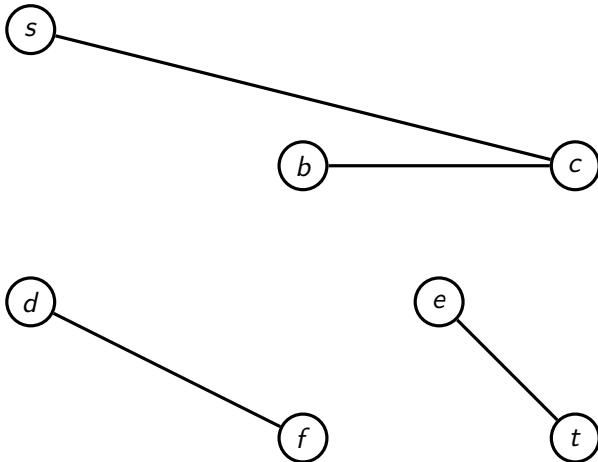
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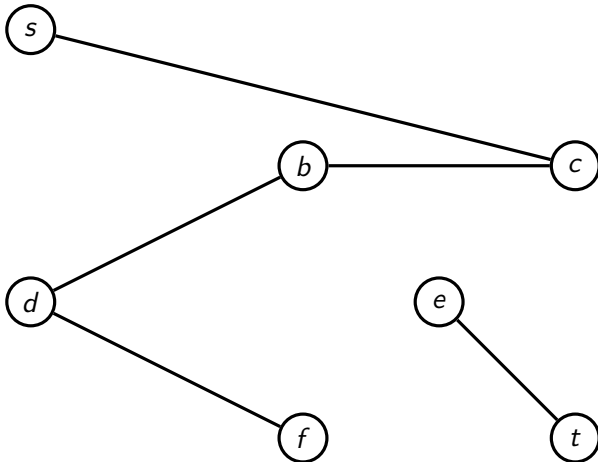
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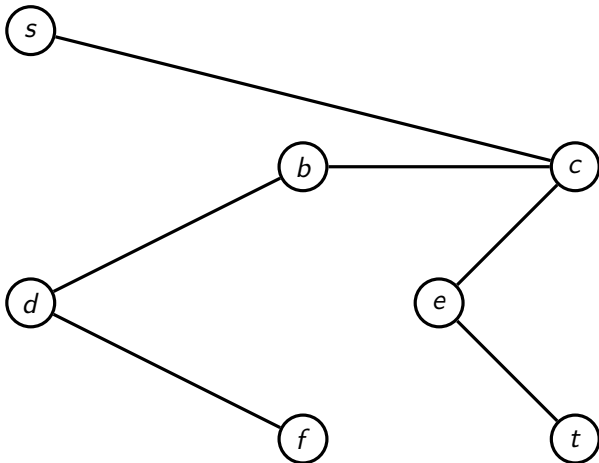
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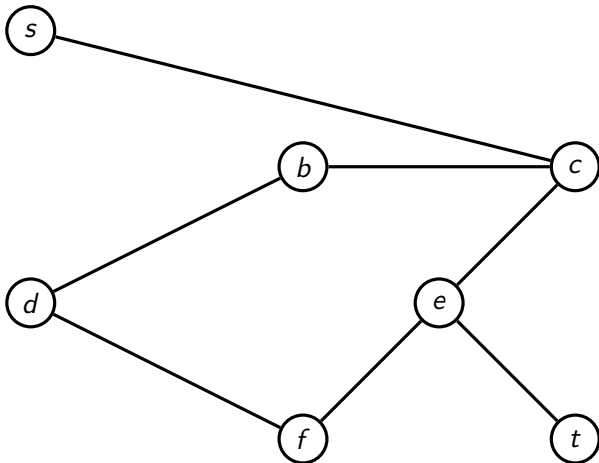
## Streaming graphs

Add edge.  $s$  is connected to  $t$ .

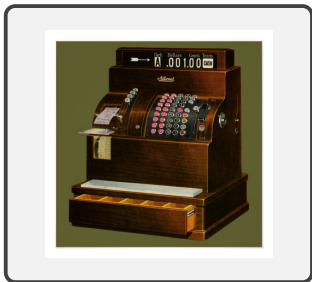


## Streaming graphs

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## The cash register and turnstile models



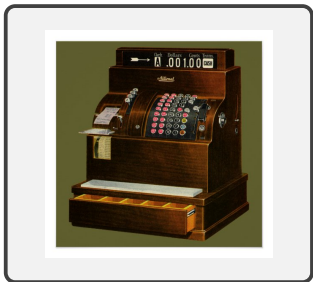
(a) cash register



(b) turnstile

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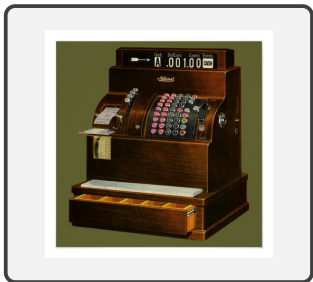
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(a) cash register



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- In the cash register streaming model counts are always non-negative.
- In the turnstile streaming the count may be negative or positive. For example we may remove copies of an IP address as well as adding copies or in a graph we may remove edges as well as add them.



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  3. Near linear time. This means near constant time per arriving token.
- If the data set is massive, fast, small space, one-pass algorithms may be needed even if it is not being streamed.

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- For example, answers that with 90% probability are within 10% of the correct value.

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## Other readings and related courses

- Andrew McGregor's 2012 course from the University of Massachusetts, Amherst (McGregor).
- Alexandr Andoni's 2015 course from the University of Columbia (Andoni).
- Indyk and Nelson's 2017 course from Harvard University (Harvard).