

# Exercise Sheet 6

## COMS10007 Algorithms 2019/2020

28.04.2020

Reminder:  $\log n$  denotes the binary logarithm, i.e.,  $\log n = \log_2 n$ .

### 1 Recurrences: Substitution Method

1. Consider the following recurrence:

$$T(1) = 1 \text{ and } T(n) = T(n-1) + n$$

Show that  $T(n) \in O(n^2)$  using the substitution method.

2. Consider the following recurrence:

$$T(1) = 1 \text{ and } T(n) = T(\lceil n/2 \rceil) + 1$$

Show that  $T(n) \in O(\log n)$  using the substitution method.

*Hint:* Use the inequality  $\lceil n/2 \rceil \leq \frac{n}{\sqrt{2}} = \frac{n}{2^{\frac{1}{2}}}$ , which holds for all  $n \geq 2$ . Use  $n = 2$  as your base case.

### 2 Search in a Sorted Matrix (difficult!)

We are given an  $n$ -by- $n$  integer matrix  $A$  that is sorted both row- and column-wise, i.e., every row is sorted in non-decreasing order from left to right, and every column is sorted in non-decreasing order from top to bottom. Give a divide-and-conquer algorithm that answers the question:

“Given an integer  $x$ , does  $A$  contain  $x$ ?”

What is the runtime of your algorithm?